Academic Approach®

SAT Math Toolkit
Passport to Advanced Math – Equivalent Expressions
PAM.EQE

Average SAT Frequency: 3.25

SAT Test Specifications

- PAM.EQE.1.a Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including rewriting simple rational expressions
- PAM.EQE.1.b Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including rewriting expressions with rational exponents and radicals
- PAM.EQE.1.c Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including factoring polynomials
- PAM.EQE.2 Fluently add, subtract, and multiply polynomials

CCSS Best Bridge

- A-APR.1 Perform arithmetic operations on polynomials. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-APR.6 Rewrite rational expressions. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-APR.7 Rewrite rational expressions. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A-SSE.2 Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
- A-SSE.3 Write expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- N-RN.2 Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Examples of Student Actions

- When adding or subtracting polynomials, start by identifying and combining like terms.
- When multiplying polynomials, first use structure to see which products of monomials will give like terms and then follow the order of operations.
- When factoring polynomials, look for common factors and/or variables that can be factored out of each term.
- When factoring polynomials, look for relationships that allow the use of the difference of two squares, the square of a binomial, and quadratic trinomials that factor into two linear terms.
- Write radical expressions in equivalent forms using fractional exponents.
- Add, subtract, multiply, and divide radical expressions.
- Use the distributive property to determine the product of a binomial and a trinomial.
- Identify the graph of a polynomial function given its equation.
- Beyond standard factoring techniques, use insight into the structure of a polynomial to factor the polynomial.
- Simplify expressions using a combination of the power, product, and quotient rules properties.

Academic Skills and Suggestions for Improvement (from Skills Insight) with Examples of Student Actions
PAM.EQE

	6-14 *(120-280)*	15-19 *(300-380)*	20-24 *(400-480)*	25-29 *(500-580)*	30-34 *(600-680)*	35-40 *(700-800)*
Academic Skills	n/a	n/a	• Use the distributive property to multiply a polynomial by either a constant or a monomial, and then combine like terms.	• Factor a monomial from a polynomial expression. • Factor a trinomial into two binomials. • Add and subtract polynomials in one variable. • Multiply two binomial expressions.	• Use properties of radicals and exponents to rewrite simple expressions. • Use properties of rational expressions to rewrite simple expressions. • Add, subtract, and multiply polynomials, using insight into the structure of the polynomial.	• Use properties of radicals and exponents to rewrite expressions. • Rewrite rational expressions, utilizing insight to recognize appropriate algebraic operations. • Factor complicated polynomial expressions using the structure of the polynomial and strategies such as repeated factoring, difference of squares, and factoring by parts.
Suggestions for Improvement	n/a	n/a	• When adding or subtracting polynomials, start by identifying and combining like terms. • When multiplying polynomials, first use structure to see which products of monomials will give like terms and then follow the order of operations. • When factoring polynomials, look for common factors and/or variables that can be factored out of each term. • When factoring polynomials, look for relationships that allow the use of the difference of two squares, the square of a binomial, and quadratic trinomials that factor into two linear terms.	• Write radical expressions in equivalent forms using fractional exponents. • Add, subtract, multiply, and divide radical expressions.	• Use the distributive property to determine the product of a binomial and a trinomial. • Identify the graph of a polynomial function given its equation. • Beyond standard factoring techniques, use insight into the structure of a polynomial to factor the polynomial.	n/a

Academic Skills and Suggestions for Improvement (from Skills Insight) with Examples of Student Actions
PAM.EQE

Examples of Student Actions	n/a	• Students simplify and write equivalent expressions that contain positive and negative exponents. • Students use their knowledge of ratios, fractions, and laws of exponents to simplify expressions.	• Students use the structure of an expression to identify ways to rewrite it. • Students use the distributive property to prove equivalency of expressions.	• Students understand that the sum, difference, and product of two polynomials produces another polynomial. • Students use the distributive property to multiply a monomial by a polynomial and understand that factoring reverses the multiplication process. • Students will factor certain forms of polynomial expressions by using the structure of the polynomials.	• Students perform addition and subtraction of rational expressions. • Students multiply and divide rational expressions and simplify using equivalent expressions. • Students explore squaring a binomial, factoring the difference of squares, and finding the product of a sum and difference of the same two terms. • Students use the quadratic formula to solve quadratic equations that cannot be easily factored.	• Students develop strategies for factoring quadratic expressions that are not easily factorable, making use of the structure of the quadratic expression. • Students solve complex quadratic equations, including those with a leading coefficient other than 1, by completing the square.

Academic Approach®

PAM.EQE SAT Exemplars

Easy – SAT TC02 Calculator

1

Which expression is equivalent to $(3x^2 + 5) - (x^2 - 2x - 3)$?

A) $x^2 + 2x + 8$

B) $2x^2 + 2x + 8$

C) $2x^2 + 7x + 3$

D) $2x^2 + 4x + 8$

Medium – SAT TH01 No Calculator

10

If $x^3 = a$ and $8x = b$, which of the following is equivalent to $a - 2b$?

A) $x(x - 2)(x + 2)$

B) $x(x - 4)(x + 4)$

C) $(x - 2)^3$

D) $(x - 4)^3$

Hard – SAT TE01 No Calculator

14

If $2x + 3y = 5$, what is the value of $4^{2x} \cdot 8^{3y}$?

A) 2^5

B) 2^{10}

C) 2^{15}

D) The value cannot be determined from the information given.

Equivalent Expressions

Use the structure of expressions to identify ways to rewrite, simplify, and prove equivalency.

1. Rewrite each rational expression as an equivalent rational expression so that all expressions have a common denominator.

   a. $\frac{3}{7}, \frac{2}{21}, \frac{1}{3}, \frac{3}{12}, \frac{8}{16}$

   b. $\frac{a}{bc}, \frac{b}{ac}, \frac{c}{ab}$

   c. $\frac{1}{(4-x)^2}, \frac{3}{(4x-3)(4-x)^2}$

   d. $\frac{3}{(x^2-x)}, \frac{5}{x}, \frac{2x+2}{(2x^2-2)}$

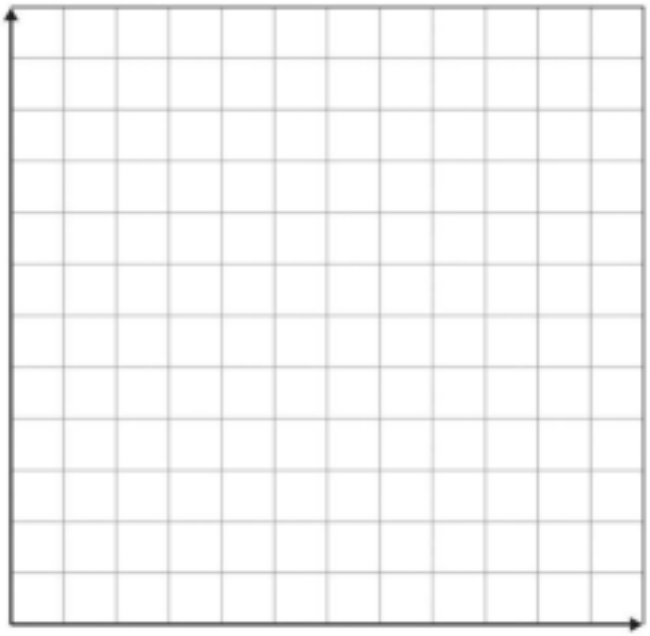
2. Use the specified methods to compare the following rational expressions: $\frac{x+3}{x^2}$ and $\frac{5+x}{2x}$.

$x =$	$\frac{x+3}{x^2}$	$\frac{5+x^2}{2x}$	Which one is larger?
1			
10			
25			
50			
100			
500			

Equivalent Expressions

Use the structure of expressions to identify ways to rewrite, simplify, and prove equivalency.

3. Graph $y = \frac{2x+3}{x}$ and $y = \frac{5}{x^2}$ for positive values of x.



4. Consider the populations of two cities where the larger city's population is M and the smaller city's population is N. For each of the following pairs, which of the expressions has a larger value? Explain your reasoning in the context of the populations.

   a. $M + N \text{ and } M$

   b. $\frac{M}{M+N}$ and $\frac{N}{M+N}$

   c. $2N \text{ and } M + N$

   d. $\frac{M}{N}$ and $\frac{N}{M}$

   e. $\frac{M}{M+N}$ and $\frac{1}{2}$

   f. $\frac{1}{M}$ and $\frac{1}{N}$