Academic Approach®

# SAT Math Toolkit
# Passport to Advanced Math – Nonlinear Equations
# PAM.NLE

*Average SAT Frequency: 6.75*

**SAT Test Specifications**

- PAM.NLE.1 Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to
    - PAM.NLE.1.a solve quadratic equations in one variable presented in a wide variety of forms. Determine the conditions under which a quadratic equation has no real solutions, one real solution, or two real solutions.
    - PAM.NLE.1.b solve simple rational and radical equations in one variable.
    - PAM.NLE.1.c identify when procedures used to solve a simple rational or radical equation in one variable lead to an equation with solutions that do not satisfy the original equation (extraneous solutions).
    - PAM.NLE.1.d solve polynomial equations in one variable that are written in factored form.
    - PAM.NLE.1.e solve linear absolute value equations in one variable.
    - PAM.NLE.1.f solve systems of linear and nonlinear equations in two variables, including relating the solutions to the graphs of the equations in the system.
- PAM.NLE.2 Given a nonlinear equation in one variable that represents a context, interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage.
- PAM.NLE.3 Given an equation or formula in two or more variables that represents a context, view it as an equation in a single variable of interest where the other variables are parameters and solve for the variable of interest.
- PAM.NLE.4 Fluently solve quadratic equations in one variable, written as a quadratic expression in standard form equal to zero, where using the quadratic formula or completing the square is the most efficient method for solving the equation.

**CCSS Best Bridge**

- 6.SP.5 Summarize and describe distributions.
- 8.F.4 Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- A-APR.3 Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.7 Rewrite rational expressions. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A-CED.4 Create equations that describe numbers or relationship. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
- A-REI.2 Understand solving equations as a process of reasoning and explain the reasoning. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- A-REI.3 Solve equations and inequalities in one variable. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unauthorized copying or reuse of any part of this page is illegal.

- A-REI.4 Solve equations and inequalities in one variable. Solve quadratic equations in one variable.
- A-REI.7 Solve systems of equations. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.
- A-SSE.1 Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context.
- A-SSE.3 Write expressions in equivalent forms to solve problems. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- F-IF.4 Interpret functions that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

**Student Actions**

- Students graphically represent a non-linear relationship between two quantities and interpret features of the graph.
- Students use the distributive property to prove equivalency of quadratic expressions.
- Students interpret word problems to create equations in one variable and solve them using factoring and the zero-product property.
- Students demonstrate knowledge that some variables in a formula can be constants that define a relationship between two or more quantities, where one is in terms of another.
- Students use appropriate and efficient strategies to find solutions to basic quadratic equations (factoring vs. using quadratic formula).
- Students rewrite quadratic expressions given in standard form, $ax^2 + bx + c$ (with $a = 1$), in the equivalent completed-square form, $a(x - h)^2 + k$, and recognize cases for which factored or completed-square form is most efficient to use.
- Students solve increasingly complex one-variable equations, some of which need algebraic manipulation, including factoring as a first step and using the zero-product property.
- Students solve complex quadratic equations, including those with a leading coefficient other than 1, by completing the square.
- Students solve cubic equations by grouping.
- Students use the discriminant, $b^2 - 4ac$, can be used to determine whether a quadratic equation has one, two, or no real solutions.

Unauthorized copying or reuse of any part of this page is illegal.

**Academic Skills and Suggestions for Improvement (from Skills Insight) with Examples of Student Actions**
**PAM.NLE**

| | 6-14 | 15-19 | 20-24 | 25-29 | 30-34 | 35-40 |
|---|---|---|---|---|---|---|
| **Academic Skills** | n/a | n/a | n/a | • Solve a quadratic equation in the form $x^2+bx+c=0$ by factoring or by using the quadratic formula.<br>• Solve a quadratic equation in the form $ax^2=b$. Solve two- and three-step radical equations in one variable.<br>• Rearrange a multivariate equation to isolate a variable or term. | • Solve multistep quadratic equations.<br>• Solve radical equations using the structure of the equation to reduce the number of algebraic steps.<br>• Solve rational equations using the structure of the equation to reduce the number of algebraic steps.<br>• Solve a system of equations consisting of one linear equation and one quadratic equation algebraically.<br>• Rearrange a multivariable equation using multiple algebraic steps to isolate a term. | • Solve quadratic, radical, and rational equations with multiple steps, where using insight into the structure of the equation provides an advantage.<br>• Determine the conditions under which a quadratic equation has zero, one, or two solutions. |
| **Suggestions for Improvement** | n/a | n/a | • Use what you know about factoring and the zero-product property to solve quadratic equations.<br>• Identify terms in quadratic equations and describe their meaning in relationship to the real-world situation that they represent. | • Identify terms in quadratic equations and describe their meaning in relationship to the real-world situation that they represent.<br>• Identify terms in polynomial equations and describe their meaning in relationship to the real-world situation that they represent.<br>• Use the inverse relationship between roots and exponents to solve equations.<br>• Apply inverse operations to solve equations.<br>• Determine whether solutions to equations are extraneous.<br>• When solving quadratic equations, determine if the most efficient method is to complete the square or to apply the quadratic formula. Look for connections between solutions of a quadratic equation and zeros of a quadratic function.<br>• Transform a quadratic equation so the x-intercepts, the y-intercept, or the maximum or minimum of its graph can easily be found by a strategically chosen form of the equation. | • Use the discriminant of a quadratic equation to determine the number of real and complex roots.<br>• Identify the graph of a polynomial function given its equation.<br>• Identify key characteristics of a quadratic equation in vertex form and standard form. | n/a |
| **Examples of Student Actions** | n/a | n/a | • Students use the distributive property to prove equivalency of quadratic expressions.<br>• Students interpret word problems to create equations in one variable and solve them using factoring and the zero-product property.<br>• Students graphically represent a non-linear relationship between two quantities and interpret features of the graph. | • Students demonstrate knowledge that some variables in a formula can be constants to define a relationship between two or more quantities, where one is in terms of another.<br>• Students use appropriate and efficient strategies to find solutions to basic quadratic equations (factoring vs. using quadratic formula).<br>• Students rewrite quadratic expressions given in standard form, $ax^2 + bx + c$ (with $a = 1$), and in the equivalent completed-square form, $a(x - h)^2 + k$, and recognize cases for which factored or completed-square form is most efficient to use. | • Students solve increasingly complex one-variable quadratic equations, some of which need algebraic manipulation, including factoring as a first step and using the zero-product property.<br>• Students solve complex quadratic equations, including those with a leading coefficient other than 1, by completing the square.<br>• Students solve cubic equations by grouping. | • Students use the discriminant, $b^2 - 4ac$, can be used to determine whether a quadratic equation has one, two, or no real solutions. |

Unauthorized copying or reuse of any part of this page is illegal.

Academic Approach®

# PAM.NLE SAT Exemplars

Easy – SAT TH01 No Calculator

**3**

What are the solutions of the quadratic equation $p^2 + 3p - 28 = 0$?

A) $p = -4$ and $p = -7$

B) $p = -4$ and $p = 7$

C) $p = 4$ and $p = -7$

D) $p = 4$ and $p = 7$

Medium – SAT TH01 Calculator

**10**

Joule's first law describes the amount of heat produced by an electric current flowing through a conductor. The amount of heat $Q$ is equivalent to the product of the square of the electric current $I$, the amount of resistance $R$ present in the conductor, and the amount of time $t$ that the current flows through the conductor. Which of the following equations gives the resistance $R$ in terms of the other variables?

A) $R = Q - I^2t$

B) $R = QI^2t$

C) $R = \frac{I^2t}{Q}$

D) $R = \frac{Q}{I^2t}$

Hard – SAT TG01 No Calculator

**18**

$$x^3 - 7x^2 + 3x - 21 = 0$$

For what real value of $x$ is the equation above true?

*E: C*
*M: D*
*H: 7*

Unauthorized copying or reuse of any part of this page is illegal.

Academic Approach®

# Nonlinear Equations

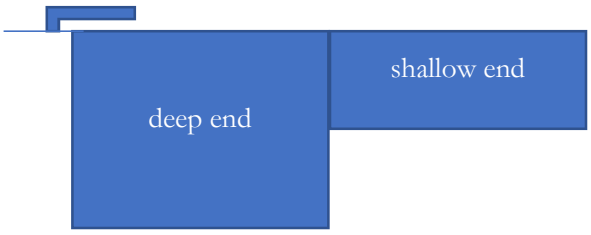
*Create, use, and interpret solutions of quadratic equations in a variety of contexts.*

Name: _____
Period: _____
Date: _____

A hotel is planning to redesign its pool to accommodate a new diving board and waterslide for guests to use. The existing rectangular pool is 28 feet long. The width of the existing pool is one foot less than three times the measure of the pool's depth.

1. The existing pool can hold up to 1,960 cubic feet of water. What are the dimensions of the existing pool? Be sure to define any variables used. Show all work.

2. To accommodate the new diving board and waterslide, the hotel management wants the length of the new rectangular pool to be 30 feet. The deep end of the new pool (where the diving board and waterslide will be placed) will be 10 feet deeper than the shallow end. The new pool's width will equal three feet more than double the depth of the shallow half. The new pool will hold 4,950 cubic feet of water. How deep will each half of the pool be? Show your work and be sure to define any variables used.




Unauthorized copying or reuse of any part of this page is illegal.